Health Monitoring of Electric Circuits

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Abstract
A methodology of defect identification in linear DC circuits based on so called *Virtual Distortion Method (VDM)* will be presented. The proposed approach takes adventage of the analogy linking mechanical models of truss structures and electric circuits. In this paper we cover the issue of modelling defects in electric circuits by compatible state of distortions, next we introduce the concept of *influence matrix* D_{ij}, which describes global sensitivity of the circuit and enables fast re-calculation of system response, and we formulate a gradient method of defect identification (inverse problem) based on local current and voltage measurements. In the following analysis we focus on so-called *impotent states* of distortions, which are the source of ambiguity of solution. We show that these difficulties can be overcome by a proper measuring strategy.

Introduction

We would like to propose a new approach to the problem of detection and identification of defects in electric circuits. This approach is based on analysis of numerical model of the circuit by the means of general system theory (global response of the system is considered). The similarities of mathematical formulations of constitutive relations and equilibrium equations allow to analyse model of electric circuit in the same manner as models of truss structures or water networks [1]. The approach presented here exploits the analogy between linear DC circuit and the static case of truss structure in the elastic range of deformations, where the state of voltages-currents related by conductance is equivalent to the state of strains-stresses related by Young modulus.

The inverse problem of defect identification in the network systems can be solved generally by a "soft computing" methods (genetic algorithms [2], neural networks) or by optimisation techniques based on full model of the system (for example gradient-based).
The defect identification problem in DC circuits will be formulated and solved according to Virtual Distortion Method [3,4]. This method is based on gradient optimisation and was originally developed to solve various issues in structural mechanic (remodelling, progressive collapse, defect identification). The main concept standing behind VDM is modelling structural modifications of the system by the field of initial deformations (so-called *virtual distortions*). Distortions generate a residual state of strains and a self-equilibrated state of stresses in the system. Superposition of the residual response and the response due to external loading gives a response of construction *modelled by distortions* which is identical in a sense of strain and stress fields with the response of construction with modified parameters. The foundation for numerical calculation in VDM is the influence matrix D_{ij} which stores the strain response of the element *i* on unit distortion imposed on the element *j*.

It is worth mentioning that similar, VDM-based approach has been already successfully utilized in the field of water networks diagnostics [5] where analogies between mechanical and hydraulic systems were also exploited.

Linear analysis of DC circuits

Let us consider the issue of solving linear DC circuits containing resistive elements and constant voltage and current sources. Assume that the circuit consists of *n* branches and *m* nodes. The topology of the circuit is described by the nodal matrix $\underline{M}_{(m-1)xn}$ (determines connections between the nodes and the branches) and the loop matrix $\underline{L}_{pxn}$ (determines elementary loops, $p = n-m+1$). Parameters of the circuit are expressed in a form of a diagonal matrix of resistances $\underline{R}$ or conductances $\underline{G}$, while the sources are ordered in a column vectors $\underline{E}$ (voltage sources) and $\underline{J}$ (current sources). Branch currents and branch voltages are represented by column vectors $\underline{I}$ and $\underline{U}$ respectively.

A response of the circuit (current and voltage distribution) to a given input can be found by applying Kirchhoff's and Ohm's laws which can be formulated as follows:

- Kirchhoff's current law: $\underline{M} * (\underline{I} + \underline{J}) = [0]$
- Kirchhoff's voltage law: $\underline{L} * (\underline{U} + \underline{E}) = [0]$
- Ohm's law: $\underline{U} = \underline{R} * \underline{I}$ or $\underline{I} = \underline{G} * \underline{U}$

Combining the above equations we obtain a general relation for the circuit response:

$$\underline{X} = (\underline{A}_X)^{-1} \cdot \underline{B} \tag{1}$$

Where: $\underline{X}$ – vector of the unknown quantity $\underline{I}$ or $\underline{U}$, $\underline{A}_x$ – matrix of parameters, $\underline{B}$ – vector of inputs. The matrices $\underline{A}_x$ and the vector $\underline{B}$ are calculated according to formulas:

$$\underline{A}_I = \left[\begin{array}{c} \underline{M} \\ \hline -\underline{L}\cdot\underline{R} \end{array}\right] \qquad \underline{A}_U = \left[\begin{array}{c} \underline{M}\cdot\underline{G} \\ \hline \underline{L} \end{array}\right] \qquad \underline{B} = \left[\begin{array}{c} -\underline{M}\cdot\underline{J} \\ \hline -\underline{L}\cdot\underline{E} \end{array}\right] \tag{2}$$

Modelling defects by distortions

We assume that a defect could be any possible change of resistance in any branch of the circuit. In the VDM formulation local modification of parameter of truss element is modelled by the initial deformation. Taking advantage of the analogy modification of the resistance in a given branch of the electric circuit can be modelled by *virtual distortion* E^0 in the form of the ideal voltage source inserted in series to the branch. The conceptual scheme is shown in Fig.1.

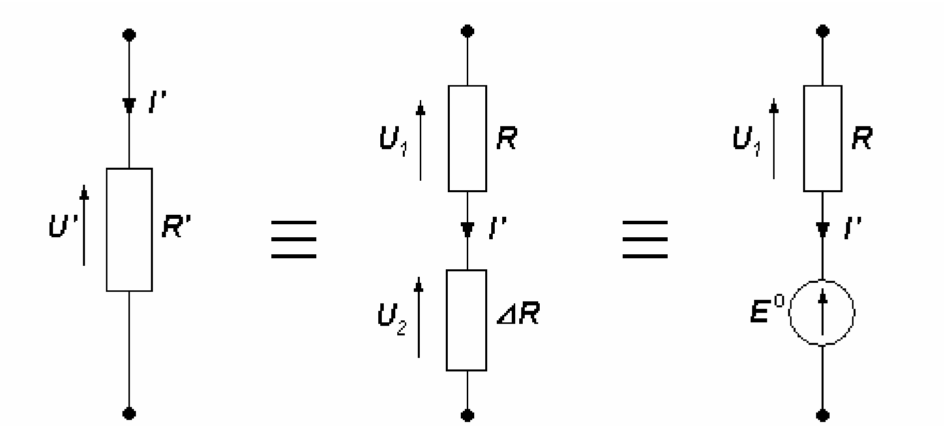

Fig. 1 - Modelling the change of resistance by the distortion

The branch with the modified resistance and the branch with the induced distortion are identical in the sense of voltage and current state if the value of the distortion fulfils the following condition:

$$E^0 = \Delta R \cdot I' = (\mu - 1) \cdot R \cdot I' = \frac{(\mu - 1)}{\mu} \cdot U' \tag{3}$$

where $\mu = R'/R$ is a modification parameter.

The above relation shows that distortions are generated only in the branches where modifications have taken place, but their values depends also on the presence of others distortions.

Linear DC circuits fulfil the superposition principle, hence the response of the circuit with the introduced distortions can be expressed as a sum of responses due to acting of real sources and responses due to acting of distortions. The response of the circuit with distortions can be obtained from Eq.1 by changing the vector of inputs $\underline{B}$ in the following manner:

$$\underline{B}' = \left[\frac{-\underline{M} \cdot \underline{J}}{-\underline{L} \cdot \underline{E}}\right] + \left[\frac{0}{-\underline{L} \cdot \underline{E}^0}\right] = \underline{B} + \underline{B}_M \cdot \underline{E}^0; \qquad \underline{B}_M = \left[\frac{0}{-\underline{L}}\right] \tag{4}$$

Inserting Eq.3 into Eq.1 we obtain the formulas:

$$\underline{I}' = \underline{I} + \underline{D}^I \cdot \underline{E}^0; \qquad \underline{U}' = \underline{U} + \underline{D}^U \cdot \underline{E}^0 \tag{5}$$

$$\underline{D}^I = (\underline{A}_I)^{-1} \cdot \underline{B}_M; \qquad \underline{D}^U = (\underline{A}_U)^{-1} \cdot \underline{B}_M + \underline{\delta} \tag{6}$$

where $\underline{\delta}$ is the identity matrix.

The matrix D^U (D^I) is called *static influence matrix.* The element D_{ij} equals to the voltage (current) induced in the i^{th} branch by unit distortion in the j^{th} branch. Both the matrices are quadratic, singular and depend only on the parameters and the topology of the initial circuit. Between D^I and D^U exists the relation:

$$\underline{D}^U - \underline{\delta} = \underline{R} \cdot \underline{D}^I \tag{7}$$

Inserting Eq.5 into Eq.3 the direct relation between the distortions and the modifications can be obtained:

$$\underline{E}^0 = -\left((\underline{\mu} - \underline{\delta}) \cdot \underline{D}^U - \underline{\mu} \right)^{-1} \cdot (\underline{\mu} - \underline{\delta}) \cdot \underline{U} \tag{8}$$

where $\underline{\mu}$ – a diagonal matrix of modification parameters.

Defect identification method

Let us consider the inverse problem of defect identification in a DC circuit. We assume that a defect can occur in any possible location, and that a system of sensors is able to measure locally voltage and current distribution (in a given branch only one quantity is measured).

Let assume that the topology, parameters and sources of the initial circuit are known. Hence, the response of the initial circuit and the influence matrices are also known (from Eq.1 and Eq.6). We will use a gradient based method to find the vector of distortions, which generates the state of voltages and currents equal to the measured one.

First, let us define the sets of functions F_k^I and F_j^U as distances between the measured responses and the responses generated by the distortions:

$$F_k^I = I_k^{meas} - I_k^{\text{mod}} = I_k^{meas} - (I_k + \sum_i D_{ki}^I \cdot E_i^0)\,; \qquad k \in \mathrm{K};\ i \in \mathrm{N} \tag{9}$$

$$F_j^U = U_j^{meas} - U_j^{\text{mod}} = U_j^{meas} - (U_j + \sum_i D_{ji}^U \cdot E_i^0)\,; \qquad j \in \mathrm{J};\ i \in \mathrm{N} \tag{10}$$

where: K – a set of branches with current measurement, J – a set of branches with voltage measurement, N – a set of all branches (a set of possible defect locations).

The objective function g is defined as the sum of the mean-square distances:

$$g = \sum_k \left(F_k^I\right)^2 + \sum_k \left(F_j^U\right)^2 \tag{11}$$

The derivative of the objective function with the respect to the distortion is given as follows:

$$\frac{\partial g}{\partial E_i^0} = 2\sum_k \left((F_k^I)\cdot(-D_{ki}^I)\right) + 2\sum_j \left((F_j^U)\cdot(-D_{ji}^U)\right) \tag{12}$$

Minimization of the objective function will be achieved using a gradient descent method. During each iteration value of distortion is updated according to the formula:

$$E_i^0(p+1) = E_i^0(p) - \lambda_p \cdot \frac{\partial g}{\partial E_i^0(p)} \tag{13}$$

where: p – the number of iteration, λ_p – a non-negative factor.

The block diagram of the iterative procedure is shown in Fig.2.

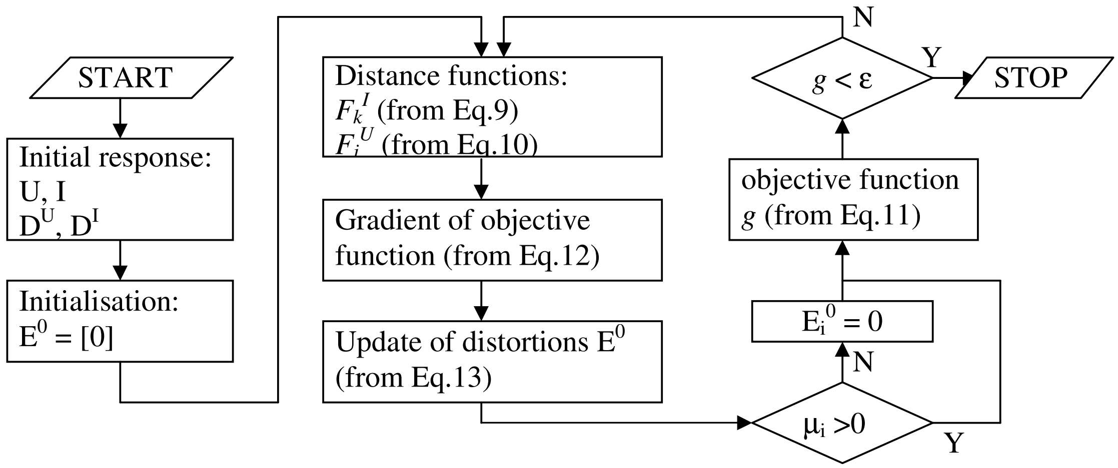

Fig.2 – A block diagram of the defect identification algorithm

To prevent the situation, when the algorithm finds unrealistic solution, the constraints on distortion evolution are introduced. The simple condition for verification that situation is μ >0. Note that from Eq.3 we obtain a very simple relation between μ and E^0:

$$\mu_i = \frac{U_i'}{U_i' - E_i^0} \tag{14}$$

Because the value of the voltage U' for a given state of distortions has to be recalculated in every iteration of the algorithm (updating function F_j^U, see Eq.10) the computational effort is negligible. The stop criterion is based on the value of the objective function.

Measuring strategy

The features of the influence matrices are the source of uniqueness of the solution, which comes from the singularity of these matrices. There exist non-zero combinations of distortions (we call them *impotent states*), which can generate a 'zero' state of currents or voltages. The circuit theory gives an explanation to this phenomena. The theorem of inserting ideal voltage sources states that the current distribution in a circuit will be preserved if the ideal voltage sources of equal values and polarity with the regard to given the node are inserted into every branch connected with this node. Moreover – the state of the voltages in all other branches will also be preserved.

Meanwhile the theorem of inserting ideal current sources states that the voltage distribution will be preserved if the ideal current sources of equal value are inserted in all branches creating a closed loop. The state of the currents will be changed only in the branches where sources were inserted. (In our case of modelling we insert a distortion of such values that the relation E^0 / R for every branch is equal).

If we depict by E^{Uo} the impotent state of distortions working in accordance to the theorem of inserting voltage sources and by E^{Io} – the impotent state of distortions working in accordance to the theorem of inserting current sources, the relations for the circuit response are as follows:

$$\begin{cases} \underline{I}' = \underline{I} + \underline{D}^I \cdot \underline{E}^0 + \underline{G} \cdot \underline{E}^{Io} \\ \underline{U}' = \underline{U} + \underline{D}^I \cdot \underline{E}^0 + \underline{E}^{Uo} \end{cases} \tag{15}$$

We see that the state E^{Io} cannot be detected by a voltage measurement, while the state E^{Uo} by a current measurement. To obtain a proper solution for any possible location of defects, it is necessary to arrange systems of sensors in such a manner as to prevent generation of all impotent states. Generation of the states E^{Io} will be suppressed if we a have sensor of the current in every independent loop of the circuit, while generation of the states E^{Uo} will be suppressed if we have a sensor of the voltage in branches connecting together all nodes of the circuit. Generation of the impotent states can be also stopped by exclusion of some branches from the set of possible defect locations (blocking distortion development). We discuss these issues in the following numerical example.

Numerical example

Let us consider the following example of a DC circuit (Fig.3). The circuit consists of 9 resistors and one constant voltage source. The topology of the given circuit can be shown in a form of *oriented graph* (Fig.4). We can distinguish two sets of branches: a tree of the graph (thick lines) and a set of closing branches (thin lines). The branches belonging to the tree connect all nodes of the circuit, but create no closed loop. Every closing branch creates a loop with branches of the tree. All nodes and branches are numbered. We use the classical convention concerning signing direction of the currents and voltages (for resistive element the sign of current is opposite to voltage). The value of the given quantity is positive if its direction is compatible with the orientation of the graph branch.

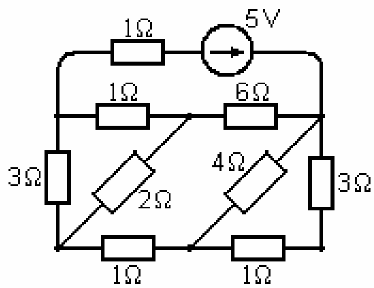

Fig.3 - Circuit diagram

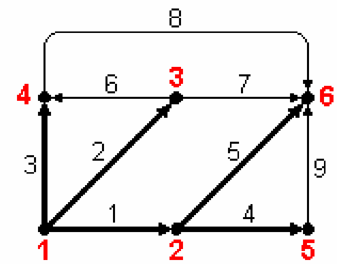

Fig.4 - Oriented graph

The matrices describing the topology of the graph have the following form:

$$\underline{M} = \left[\begin{array}{ccccc|cccc} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \qquad \underline{L} = \left[\begin{array}{ccccc|cccc} 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Ordering resistances into a diagonal matrix $\underline{R}$ and the sources into a column vector $\underline{E}$ (indexes equal to the branch numbers), from Eq.2 and Eq.1 we obtain the following response of the circuit:

$\underline{I}$ = [-0,7755 0,3061 0,4694 -0,3878 -0,3878 0,7959 -0,4898 1,2653 -0,3878]^T

$\underline{U}$ = [0,7755 -0,6122 -1,4082 0,3878 1,5510 -0,7959 2,9388 -1,2653 1,1633]^T

The voltage influence matrix derived from Eq.6:

$$\underline{D}^U = \begin{pmatrix} 0.8082 & 0.2041 & 0.2694 & -0.0959 & -0.3837 & 0.0653 & 0.2204 & 0.1551 & -0.2878 \\ 0.1020 & 0.5510 & 0.3673 & 0.0510 & 0.2041 & -0.1837 & -0.2449 & -0.0612 & 0.1531 \\ 0.0898 & 0.2449 & 0.3633 & 0.0449 & 0.1796 & 0.1184 & 0.0245 & -0.0939 & 0.1347 \\ -0.0959 & 0.1020 & 0.1347 & 0.8270 & 0.3082 & 0.0327 & 0.1102 & 0.0776 & -0.5189 \\ -0.0959 & 0.1020 & 0.1347 & 0.0770 & 0.3082 & 0.0327 & 0.1102 & 0.0776 & 0.2311 \\ 0.0653 & -0.3673 & 0.3551 & 0.0327 & 0.1306 & 0.7224 & 0.5633 & -0.1592 & 0.0980 \\ 0.0367 & -0.0816 & 0.0122 & 0.0184 & 0.0735 & 0.0939 & 0.1918 & 0.0980 & 0.0551 \\ 0.1551 & -0.1224 & -0.2816 & 0.0776 & 0.3102 & -0.1592 & 0.5878 & 0.7469 & 0.2327 \\ -0.0959 & 0.1020 & 0.1347 & -0.1730 & 0.3082 & 0.0327 & 0.1102 & 0.0776 & 0.4811 \end{pmatrix}$$

Let us introduce the following modifications into the circuit:

$$\mu_1 = 10; \quad \mu_6 = 5; \quad \mu_9 = 0.333 \qquad (16)$$

The vector of distortion modelling the above modifications calculated from Eq.8:

$$\underline{E}^0 = [2{,}4070 \quad 0 \quad 0 \quad 0 \quad 0 \quad -1{,}2328 \quad 0 \quad 0 \quad -0{,}3566]^T \qquad (17)$$

The response of the modified circuit (with modifications modelled via virtual distortions (17)) can be computed from Eqs. (5):

$\underline{I}'$ = [-0,2674   -0,1477   0,4152   -0,1783   -0,0891   0,3082 -0,4559   0,7234   -0,1783]^T

$\underline{U}'$ = [2,6745   0,2955   -1,2455   0,1783   0,3566   -1,5410   2,7356   -0,7234   0,1783]^T

It can be demonstrated that the same response $\underline{I}'$, $\underline{U}'$ are generated in the modified network (with the matrix $\underline{R}$ modified according to the conditions (16)), but without any distortions imposed.

As mentioned before, the proper measuring strategy has to be applied to avoid generation of the impotent states of distortions during the optimisation process. Having the graph of the circuit with the tree distinguished and closing branches, the measuring strategy resolve to a simple rule: we have to put the current sensors into all closing branches and the voltage sensors into all branches of the tree. The way of selecting the tree and closing branches is obviously arbitral, so the arrangement shown in Fig.4 is only one of many possible combinations.

We have carried out the iterative procedure of defect identification (See Fig.2) for the circuit with modified parameters, according to the declared measuring strategy. The evolution of the distortions is presented in Fig.5.

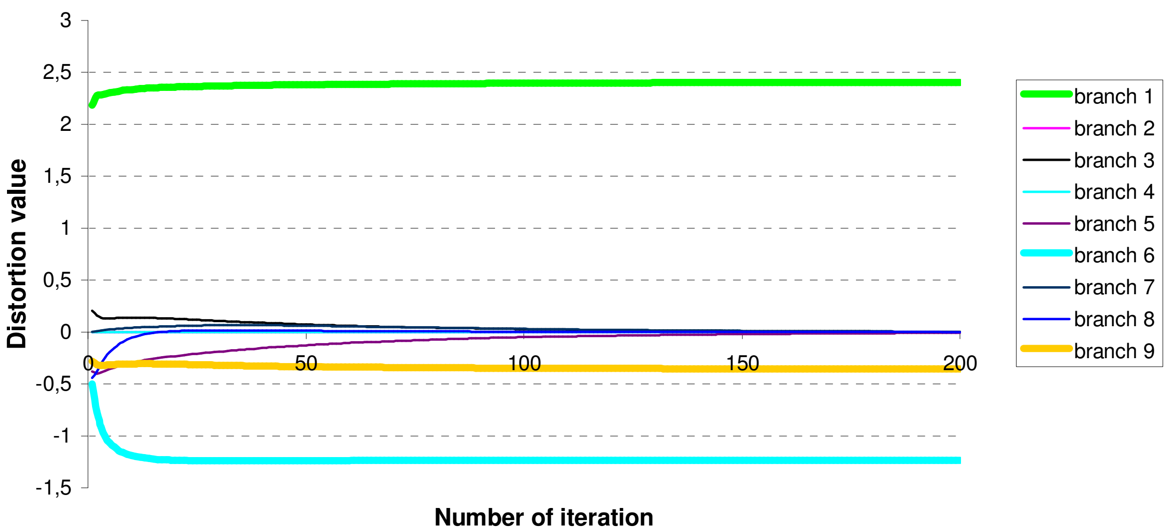

Fig.5 – Distortions development during the identification procedure

After 200 iterations the value of the objective function fell from $g_{(0)} = 6,5$ to $g_{(200)} = 2,2*10^{-8}$, and we obtained the following vector of distortions:

$\underline{E}^0$ = [2,4069   0,0002   0,0002   0,0001   0,0010   -1,2328   0,0007   0,0001   -0,3564]^T

As one can see the result is almost identical with the correct solution (17).

Conclusions

It has been demonstrated that the methodology of VDM can be successfully applied in a field of linear DC circuits. We have introduced the basic formulations and we have presented a solution to the inverse problem of defect identification.

In the future studies we want to expand the proposed methodology to AC circuits. We would like also to explore the possibility of application the VDM methodology to dynamic cases. The ultimate goal of our work is formulation the foundations of monitoring system based on the analysis of un-steady states in electric circuits caused by impulse input functions (the analogy to health monitoring systems based on the elastic wave propagation). We hope that these approaches will result in significant reduction of the number of sensors required in the measuring process.

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